

ADAPTIVE TOTAL VARIATION DERINGING METHOD FOR IMAGE INTERPOLATION

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ABSTRACT

This paper presents a new adaptive post-processing algorithm for ringing artifact reduction after image interpolation (upsampling). The algorithm is based on the concept of total variation (TV) for ringing control. It uses known TV of the blocks of the low-resolution image. Conditional gradient, subgradient and projection subgradient methods for this algorithm are considered and analyzed. A test set of 181300 overlapping 11x11 blocks of real images was used for local algorithm optimization and analysis. Local conditional gradient method shows the best objective and subjective results.

Index Terms— deringing, total variation, image interpolation, image enhancement

1. INTRODUCTION

Existing image deringing algorithms are usually focused on ringing reduction after specific resampling or compression algorithms and do not use information on the downsampled or undisturbed image. They can be divided into the following large groups:

1. Algorithms that suppress ringing and block effect for specific compression algorithms. For example, methods based on partial differential equations are used for JPEG2000 deringing [1]. JPEG blocking and ringing effects are reduced by filtering in DCT domain [2], or by specific artifacts removing algorithms [3].

2. Algorithms using knowledge of the structure of image or using edge detection. The example is cartoon deringing using example-based synthesis [4].

3. General filtering algorithms like median filtering [5], diffusion [6], deconvolution, regularization [7], or edge enhancement [8]. Ringing suppression in this case is not a primary goal and it results in image oversmoothing.

In this paper we design an adaptive image deringing (Gibbs effect suppression) algorithm for the case of image post-processing after image interpolation (upsampling). The method is based on the concept of TV for ringing control. It uses known TV of the blocks of the low-resolution image.

Originally, TV method in image processing literature was proposed by Rudin, Osher and Fatemi [9] where TV

image denoising was considered. Gibbs phenomenon (ringing) is also associated with image TV [10].

In two-dimensional case, TV can be defined in different ways.

1. $TV(z) = \iint_{\Omega} \|\nabla z\| d\Omega = \iint_{\Omega} \sqrt{|z_x|^2 + |z_y|^2} d\Omega$,
2. $TV(z) = \iint_{\Omega} (|z_x| + |z_y|) d\Omega$.

The first variant was used by Rudin, Osher, Fatemi, but many image processing algorithms use the second one [11]. We use the second variant.

General TV minimization problem can be posed in 3 forms.

1. $z = \arg \min_{z \in Z} (\|Az - u\| + \alpha \cdot TV(z))$,
2. $z = \arg \min_{z \in M} \|Az - u\|$, $M = \{z \in Z \mid TV(z) \leq C\}$,
3. $z = \arg \min_{z \in M} TV(z)$, $M = \{z \in Z \mid \|Az - u\| \leq \delta\}$,

where A is a continuous linear operator. The first problem is a particular case of Tikhonov regularization method, and this problem is well-posed [12], because $TV(z)$ is a convex functional. The second one is a particular case of a problem of finding quasi-solutions [13]. The set M is convex, and the obtained problem is well-posed under additional condition that z lie in a bounded set. This is true in the case of $A = E$ (unit operator). The third problem is also well-posed. Using the approach [14] we have proved the equivalence of these forms in the case $A = E$. Thus for any C there are α and δ , so the solution of the first problem is also the solution of the second and the third problems and vice versa.

2. PROBLEM DEFINITION

First we consider one-dimensional TV problem with $A = E$ in discrete form for an initial signal v and upsampled signal u with scale factor s . TV of high-resolution signals and images is usually greater than their low-resolution variants. Many real objects have fractal nature and there are fractal-based resampling algorithms [15], but most of the resampling algorithms do not use this fact. We assume, that TV does not change, and additional variation of upsampled

image is caused by ringing artifacts. So, if $TV(u) > TV(v)$, then we solve the following variational problem for 1-D:

$$z = \arg \min_{z \in M} \|z - u\|_2, \text{ where}$$

$$M = \{z \in Z \mid TV(z) \leq C = TV(v)\}, \quad TV(z) = \sum_i |z_i - z_{i-1}|.$$

In two-dimensional case, we assume, that the variation of any single row or column does not change, but the number of rows and columns is proportional to the scale factor s . So, the problem of image deringing can be considered as

$$z = \arg \min_{z \in M} \|z - u\|_2, \text{ where}$$

$$M = \{z \in Z \mid TV(z) \leq C = s \cdot TV(v)\},$$

$$TV(z) = \sum_{i,j} |z_{i,j} - z_{i,j-1}| + \sum_{i,j} |z_{i,j} - z_{i-1,j}|.$$

We pose two problems: standard (global) TV problem for entire image and local block TV variation problem.

2.1. Global deringing

We consider interpolated image u as a function of a Hilbert space H with Euclidian norm and scalar product. The set of images with bounded TV $M = \{z \in Z \mid TV(z) \leq C = TV(v)\}$ is a convex set in H . We find Euclidian projection of u onto M :

$$z = \arg \min_{z \in M} \|z - u\|_2, \text{ where} \quad (1)$$

$$M = \{z \in Z \mid TV(z) \leq C = s \cdot TV(v)\}.$$

This algorithm can also be used for general deringing, when initial image is unknown. In this case, we define TV reduction factor $k \in [0, 1]$ and solve the problem

$$z = \arg \min_{z \in M} \|z - u\|_2, \quad M = \{z \in Z \mid TV(z) \leq C = k \cdot TV(v)\}.$$

This problem can be also solved by regularization method:

$$z = \arg \min_{z \in Z} (\|z - u\|_2 + \alpha \cdot TV(z)), \quad (2)$$

where $\alpha = \alpha(C)$ is a regularization parameter. The problem (1) is equivalent to (2).

2.2. Local deringing

Images are not uniform, and ringing artifacts are not uniformly distributed and the method, which reduces TV for the whole image is not effective. Ringing artifacts should be removed locally. We divide both initial and interpolated image into a set of corresponding square blocks with equal size. Each pair of blocks is processed by global deringing algorithm separately. Overlapping blocks are taken to eliminate blocking artifacts.

The size of these blocks is chosen in accordance to the size of ringing effect. It depends on used interpolation algorithm. For example, for interpolation by 2, we analyzed

blocks from 7x7 (4x4 in initial image) to 15x15 in interpolated image (8x8 in initial image). Small size results in bad ringing suppression, big block size give loss of fine details. Our test results were practically the same for block size in the region from 7x7 to 15x15.

3. NUMERICAL METHODS ANALYSIS

There are several effective iterative methods to solve problems (1) and (2). We consider conditional gradient method and subgradient projection method for solving (1), and subgradient method for solving (2). A set of 100 nature and architecture images with 400x300 resolution was used to test the methods. We downsampled the images by 2 using Gauss blur with radius 0.7 and then upsampled them by regularization-based interpolation algorithm [16]. Next we applied deringing method and compared the results with initial images (reference images). Below we show the results for overlapping 11x11 blocks (1813 per image) and for the global method.

3.1. Conditional gradient method

Conditional gradient method for minimization of convex functional $F(z) = \|z - u\|_2^2$ in a convex polyhedron was introduced in [17]. The main idea of this method is to find a minimizing direction from a finite set of vectors [18]. The vertices of the convex set $M = \{z \mid TV(z) \leq C\}$ can be written easily only for one-dimensional case and we use one-dimensional algorithm to process two-dimensional images. It is applied separately to a set of corresponding rows and columns of the source and interpolated images.

The following ways of constructing the resulting two-dimensional image by the algorithm were analyzed:

1. We process the interpolated image horizontally (by rows) $z \rightarrow z_H$. This means that each row of image z is processed independently from others, and the processed rows form image z_H . Next we process the interpolated image vertically (by columns) $z \rightarrow z_V$, and calculate the weighted sum between these resulting images $z_R = \alpha z_H + (1 - \alpha) z_V$, $\alpha \in [0, 1]$.

2. We compute two resulting images: the first one is processed horizontally and then vertically $z \rightarrow z_H \rightarrow z_{HV}$, and the second one is processed vertically and then horizontally $z \rightarrow z_V \rightarrow z_{VH}$. The final image is calculated as $z_R = \alpha z_{HV} + (1 - \alpha) z_{VH}$, $\alpha \in [0, 1]$. This is twice slower than the first way.

Weight value α can also be chosen differently:

1. $\alpha = 0.5$.
2. The weight α is chosen in accordance with minimization the discrepancy $\|z_R - u\|_2$. It is correlated with

the optimal coefficient α_{opt} , which minimizes the discrepancy between the resulting image and reference image $\|z_R - \bar{u}\|_2$. For the test image set $cor(\alpha, \alpha_{opt}) = 0.27$.

3. Choose α equal to 0 or 1 by analysis of the block structure. This way is twice faster than first two, because we need to calculate only single result z_H or z_V (z_{HV} or z_{VH}), but not both. We use the following algorithm: if horizontal TV suppression value is greater than vertical $TV_H(u) - s \cdot TV_H(v) > TV_V(u) - s \cdot TV_V(v)$, where

$$TV_H(u) = \int_{\Omega} |u_x| d\Omega, \quad TV_V(u) = \int_{\Omega} |u_y| d\Omega,$$

then we use $\alpha = 1$, else $\alpha = 0$.

The average PSNR test set results for 200 iterations are

	1st way	2nd way
no processing	28.38	
$\alpha = 0.5$	29.15	29.24
$\alpha \in [0, 1]$	29.00	29.23
$\alpha = 0$ or $\alpha = 1$	29.05	29.22

Table 1. 2-D processing parameter analysis results.

The second way of calculating z_R and $\alpha = 0.5$ gives the best results. However, the third way of choosing α can be used if we need to accelerate the algorithm.

3.2. Subgradient method

Generally, the subgradient method looks like follows [19]:

$$z^{(k+1)} = z^{(k)} - \alpha_k g^{(k)}, \quad g^{(k)} \in \partial F(z^{(k)}),$$

where $g^{(k)}$ is any subgradient of $F(z)$ in $z^{(k)}$, α_k are the step lengths. $g^{(k)}$ is an element of subgradient set $\partial F(z^{(k)})$ of $F(z)$ in $z^{(k)}$ if it satisfies the condition $F(z) \geq F(z^{(k)}) + (g^{(k)}, z - z^{(k)})$ for all z . We use this method for minimizing functional (2) with $\alpha_k = \alpha_0 \gamma^k$.

This method is effective in two-dimensional case as well as in one-dimensional, but the disadvantage is that it needs calculation of regularization parameter α , while the function $\alpha = \alpha(C)$ is unknown. The function $f(x) = f(\log \alpha) = C(\alpha)$ is non-increasing and close to linear function. We estimate $x = \log \alpha$ by using dichotomy method. However, it requires to run the minimization of regularization functional (2) for several different α , so the subgradient method is quite slow.

3.3. Projection subgradient method

Projection subgradient method is used instead of simple subgradient method if a problem has a constraint $z \in M$. It

is similar to subgradient method and it can be formulated in the following form [19]:

$$z^{(k+1)} = pr_M(z^{(k)} - \alpha_k g^{(k)}), \quad (3)$$

where pr_M is the projection operator onto convex set M . If the set M is defined as $M = \{z | f(z) \leq 0\}$, where $f(z)$ is a convex functional, then the projection operator can be approximated iteratively. In our case $F(z) = \|z - u\|_2^2$, $f(z) = TV(z) - C$, and the solution of minimization problem is equal to the projection of u onto the set M , or just performing one iteration of (3) with $z^{(0)} = u$. This approach is also used for image restoration in [20]. The projection can be calculated approximately as $z^{(k+1)} = z^{(k)} - \alpha_k g^{(k)}$, where

$$g^{(k)} = \begin{cases} \partial f(z^{(k)}), & z^{(k)} \notin M, \\ 0, & z^{(k)} \in M. \end{cases}$$

3.4. Methods summary

We have analyzed the behavior of these methods in one-dimensional case for 50 iterations.

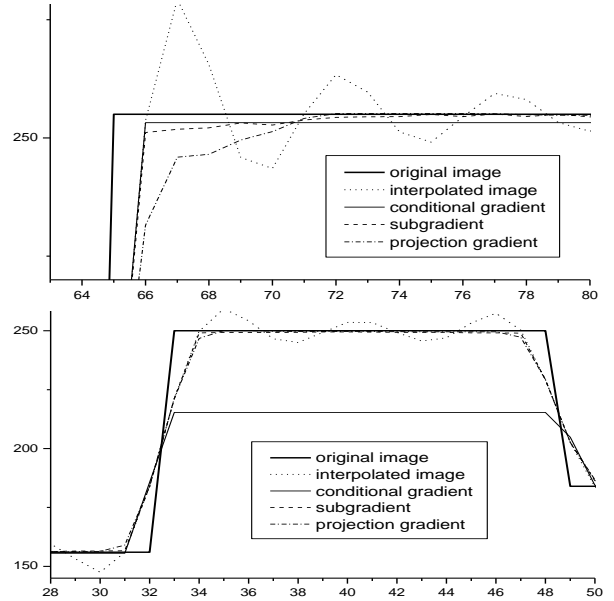


Fig. 1. Numerical methods comparison in 1-D case.

Conditional gradient method shows good results on simple input data like step function, but it convergences very slowly on complex data even for bar function. So, it can be effectively used only for processing of small blocks.

Regularization (subgradient) method (2) provides good results, but it requires pre-calculating regularization parameter, so this method is quite slow.

Both conditional and subgradient methods converge to the solution of (1) if number of iterations tends to infinity.

One-step projection gradient method is effective, but it never converges to the solution of (1). This method

smoothes edges but if the Gibbs phenomenon is low, then the difference between interpolated image and optimal image is small, and the error between optimal image and the result of projection gradient method is very low.

Next, we have processed the test image set by these methods and calculated average PSNR values, which are shown in table 2. The results were different for different images. Note that subgradient method for global deringing is a solution for pure TV minimization problem (1).

	Global deringing	Local deringing
No processing	28.38	
Conditional gradient method (2nd way, $\alpha = 0.5$)	23.33	29.24
Fast conditional gradient method(2nd way, $\alpha = 0$ or 1)	22.17	29.22
Subgradient method	28.71	28.64
Projection subgradient	28.75	28.68

Table 2. Results for test image set.

Image upsampling result by a modification of Lanczos interpolation method and post-processed image by the proposed method (2nd way conditional gradient method with $\alpha = 0.5$) is given in Fig. 2.

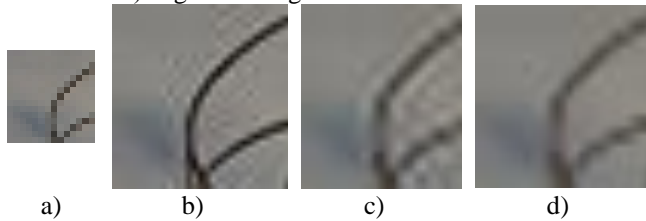


Fig. 2. Method results: a) low-resolution image; b) reference image; c) interpolated image; d)deringed image.

4. CONCLUSION

New image deringing method for image interpolation using information on the TV from initial image has been proposed. It was found that local approach with conditional gradient method shows better results than pure TV approach. Experiments show high efficiency of the method for resampling post-processing. The corresponding free resampling software is available at <http://imaging.cs.msu.ru/software>. The work was supported by RFBR grant 06-01-39006-ГФЕН_a.

5. REFERENCES

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