IMAGE SUPER-RESOLUTION USING FAST DECONVOLUTION[1]

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Super-resolution problem is posed as an inverse deconvolution problem. Fast non-iterative super-resolution algorithm based on this approach is suggested.

Introduction

The problem of super-resolution (SR) is to recover a high-resolution image from a set of several degraded low-resolution images. This problem is very helpful in human surveillance, biometrics, etc. because it can significantly improve image quality.
There are two groups of video SR algorithms: learning- and reconstruction-based. Learning-based algorithms enhance the resolution of a single image using information on the correspondence of sample low- and high-resolution images. Reconstruction-based algorithms use only a set of low-resolution images to construct high-resolution image. More detailed introduction into video SR problems is given in [1], [2].
The majority of reconstruction-based algorithms use camera models [3] for downsampling the high-resolution image. The problem is posed as error minimization problem

$$z_R = \arg\min_z \sum_{k=1}^{N} \left\| A_k z - w_k \right\|_2^2 , \qquad (1)$$

where z is reconstructed high-resolution image, w_k is k-th low-resolution image, A_k is a downsampling operator which transforms high-resolution image into k-th low-resolution image, $\|\cdot\|_2$ is standard Euclidian norm. The operator can be generally represented as $A_k z = DH_{cam}F_k H_{atm} z + n$, where H_{atm} is atmosphere turbulence effect which is often neglected, F_k is a warping operator like motion blur or motion deformation, H_{cam} is camera lens blur which is usually modeled by Gauss filter, D is a decimation operator, n is a noise which is usually ignored. For computational purposes, we exchange $H = H_{cam}$ and F_k and use

$$A_k z = DF_k Hz . \qquad (2)$$

Warping operator F_k can be calculated, for example, using motion calculation at base points and interpolation at other points [4], [5]. Variational optical flow estimation approaches are also widely used [6], [7], [8], [9].

Problem definition

Since z is defined on a discrete set, but pixel coordinates $(x^*, y^*) = F_k(x, y)$ do not always belong to the grid, operator H is used both for filtering and interpolation:

$$Hz(x^*, y^*) = \frac{\sum\limits_{(x_i, y_j)} z(x_i, y_j) e^{-\frac{(x^* - x_i)^2 + (y^* - y_j)^2}{2\sigma^2}}}{\sum\limits_{(x_i, y_j)} e^{-\frac{(x^* - x_i)^2 + (y^* - y_j)^2}{2\sigma^2}}} , \qquad (3)$$

where (x_i, y_j) are grid points, σ is chosen in accordance with scale factor s. We use $\sigma = 0.4s$. Operator D simply rescales the coordinates.
Operator $v_k = A_k z$ (2) takes the form

$$v_k(x_i, y_j) = Hz(x_{i,j}^k, y_{i,j}^k) , \qquad (4)$$

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where (x_i, y_j) are grid points, $(x_{i,j}^k, y_{i,j}^k) = F_k(sx_i, sy_j)$. The super-resolution problem (1) takes the form

$$z_R = \arg\min_z \sum_{i,j,k} \left| Hz(x_{i,j}^k, y_{i,j}^k) - w_k(x_i, y_j) \right|^2 . \quad (5)$$

By changing multiple indexes with single index, the formula (5) can be rewritten as

$$z_R = \arg\min_z \sum_{n=1}^{N} \left| Hz(x_n, y_n) - w_n \right|^2 \quad (6)$$

The problem (6) is ill-posed, so regularization methods [10] are used:

$$z_R = \arg\min_z \left(\sum_{n=1}^{N} \left| Hz(x_n, y_n) - w_n \right|^2 + \alpha f(z) \right). \quad (7)$$

Iterative method for solving (7) is discussed in [2]. In this paper, a non-iterative algorithm for solving (6) is proposed.

Deconvolution

We consider the problem of deconvolution on a discrete 1D set $\Omega = \{x_i : x_i = ih\}$ and the case of Gauss filter G

$$G_i = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x_i^2}{2\sigma^2}\right). \quad (8)$$

The convolution looks as

$$y = Hz = z * G, \quad y_i = \sum_j z_j G_{i-j} . \quad (9)$$

The problem of deconvolution is to reconstruct z from given convolution result Hz

$$z = H^{-1} y . \quad (10)$$

Inverse operator H^{-1} can be constructed using Fourier transform: $\hat{y} = \hat{z}\hat{G}$, $\hat{z} = \hat{y} / \hat{G}$, and z can be found as a convolution of y with inverse Fourier transform of $1/\hat{G}$. Operator H^{-1} is unbounded. In the case of noisy data it significantly amplifies noise. To avoid this, we use a finite adaptive filter

$$z = y * C, \quad z_i = \sum_{j=-k}^{j=k} z_j c_{i-j}, \; c_j = c_{-j} . \quad (11)$$

Coefficients c_j in (11) are chosen to minimize $\| z - y * C \|_2$. Filter length k is chosen in a way to make deconvolution fast, but precise enough. We use $k = 3$.

In two-dimensional case, we process consequently the rows and the columns of the image.

For given super-resolution problem (1), we convolve low-resolution images with Gauss filter and calculate coefficients c_j from a given set of images. We seek for

$$\{c_j\} = \arg\min_C \sum_{k=1}^{N} \| w_k - w_k * G * C \|_2^2 . \quad (12)$$

Experiments have shown that adaptive filter (11) does not significantly amplify noise. It depends on given images. If the images are noisy, then filter coefficients are smoothed and noise level does not significantly increase after deconvolution. This also means that regularization term (7) is not necessary due to adaptive filter (11) is automatically tuned to noise level.

We have compared adaptive filter with unsharp mask $z = \alpha(y - y * G) + y * G$. Unsharp mask shows practically the same results, but it takes more time to estimate its parameters (α, σ).

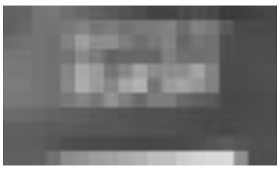
Problem solution

If the points (x_n, y_n) in (6) are grid points, then deconvolution method using adaptive filter can be used. But in general case coordinates x_n, y_n are not discrete. So, the algorithm becomes as follows:

1. Estimate the values of Hz at all grid points (x_i, y_j).
2. Perform deconvolution using adaptive filter.

To estimate the values of Hz at grid points, we use Gauss interpolation (3) with a small enough radius, so it does not significantly influences deconvolution.

In Figure 1, a result of the proposed super-resolution method is compared with other image resampling and super-resolution methods.



a) One of the input images interpolated using nearest neighbor method

b) Regularization-based super-resolution [2]

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c) The proposed super-resolution method (6) e) Regularization-based image interpolation [11]

Fig. 1. Super-resolution results using 4 input images and scale factor *s*=4.

Problem discussion

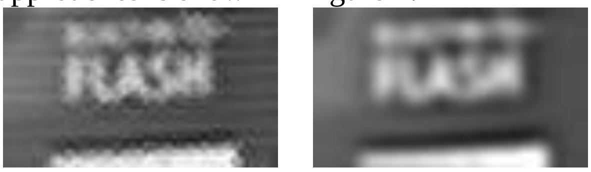
Super-resolution method (6) shows very good results if the warping operator F_k is calculated precisely. If it has errors, the solution becomes unstable. To avoid this, we can formulate super-resolution problem (1) in another way:

$$z_R = \frac{1}{N}\sum_{k=1}^{N} \arg\min_{z} \left\| A_k z - w_k \right\|_2^2 . \qquad (13)$$

We make single-image super-resolution for every image and then calculate an average image.

Using the approach (13) results in blurred image, but without artifacts caused by the inaccuracy in warping operator determining.

An example of the comparison of the approaches is shown in Figure 2.



a) The proposed super-resolution method (6) b) Super-resolution for unstable data (13)

Fig. 2. Super-resolution results for 4 input images and factor *s*=4 for inaccurately defined warping operator.

Conclusion

Fast non-iterative method for image super-resolution has been suggested. The method shows very good results if the warping operator is accurately estimated like in the case of only sub-pixel shifts in the initial image set. Hybrid method to suppress the problems of warping operator inaccurate determining is under work.

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